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EXPLICIT CONSTRUCTION OF $N=2$ W_3 CURRENT IN THE $N=2$ COSET $\frac{SU(3)}{SU(2)\times U(1)}$ MODEL

Changhyun Ahn*

Bogoliubov Theoretical Laboratory,
JINR, Dubna, Head Post Office, P.O.Box 79,
101 000, Moscow, Russia

ABSTRACT

We discuss the nonlinear extension of $N=2$ superconformal algebra by generalizing Sugawara construction and coset construction built from $N=2$ current algebra based on Kazama-Suzuki $N=2$ coset model $\frac{SU(3)}{SU(2)\times U(1)}$ in $N=2$ superspace. For the generic unitary minimal series $c=6(1-\frac{3}{k+3})$ where k is the level of $SU(3)$ supersymmetric Wess-Zumino-Witten model, this algebra reproduces exactly $N=2$ W_3 algebra which has been worked out by Romans in component formalism.

*email: ahn@thsun1.jinr.dubna.su

In recent years important progress has been made in understanding the structure of nonlinear extension of (super) conformal algebra in two dimensional rational conformal field theory. There are three approaches [1] which can be used to investigate extended Virasoro symmetries, W symmetries. One of them, third approach, is to begin with the Wess-Zumino-Witten (WZW) conformal field theory and to construct extra symmetry currents from the basic fields taking values in the underlying finite dimensional Lie algebra in that model.

In this approach, a generalization of Sugawara construction, Casimir construction, which includes higher spin generator besides the stress energy tensor in terms of currents was first presented in [2]. These analysis for level-1 WZW models provided W algebras associated with the simply laced classical Lie algebras ADE in which the application of current algebra played an important role. Furthermore, the extension [3] to a coset [4] construction led to study the unitary minimal models for the bosonic W_3 algebra with the central charge given by $c_{N=0} = 2\left[1 - \frac{12}{(k+3)(k+4)}\right], \quad k = 1, 2, \ldots$

From the fact that the appropriate extended algebra for a specific series of $A_2^{(1)}$ coset models of level $(3, k)$ is an extension of $N = 1$ Virasoro algebra and the first model in this series, with level $(1, k)$ has bosonic W_n symmetry, it has been developed further in the case of $N = 1$ W_3 algebra [5] that for $c_{N=1} = 4\left[1 - \frac{18}{(k+3)(k+6)}\right], \quad k = 1, 2, \ldots,$ the complete set of supercurrents of it should contain eight supercurrents. Of course the only five supercurrents were constructed by computing the operator product expansions (OPE's) explicitly and came up with final analysis using so-called 'character technique'.

It is a natural question to ask for $N = 2$ W_3 case. It was shown [6] that a large class of unitary conformal field theories with $N = 2$ superconformal symmetry can be realized as coset model $\frac{G}{H}$ called hermitian symmetric spaces. These $N = 2$ model based on $\frac{SU(3)}{SU(2)\times U(1)}$ has an $N = 2$ W_3 algebra [7] as its chiral algebra by duality symmetry of compact Kazama-Suzuki models and has a central charge,

$$c_{N=2} = 6\left(1 - \frac{3}{k+3}\right), \quad k = 1, 2, \ldots \tag{1}$$

It is very instructive to consider these minimal models in the context of $N = 2$ supersymmetric extension of the affine Lie algebra that was given [8] in $N = 2$ superspace in terms of supercurrents satisfying nonlinear constraints. We would like to apply this supercurrent algebra to supersymmetric WZW conformal field theory and understand how higher spin 2 supercurrent appears in $SU(3)$ $N = 2$ affine Lie algebra.

In this paper, we make an attempt to identify the independent generating supercurrents in terms of the basic superfields satisfying the nonlinear constraints through a generalized Sugawara construction and coset construction, as we will see below, in $N = 2$ CP_2 model in the above series (1).

Let us first consider a few things about the on-shell current algebra [8, 9] in $N = 2$ superspace for the supersymmetric WZW model, with level k, on a group $G = SU(3)$. We choose a complex basis for the Lie algebra, labelled by a, $\bar{a}$, $a = 1, 2, \ldots, \frac{1}{2}\dim G (=$ 4 in the adjoint representation of G), in which the complex structure related to the second supersymmetry has eigenvalue $+i$ on the hermitian generators T_a and $-i$ on the conjugated generators $T_{\bar{a}} (= T_a^{\dagger})$. In this complex basis, they satisfy the following relations: $[T_a, T_b] =$

$f_{ab}{}^{c}T_c$, $[T_a, T_{\bar{b}}] = f_{a\bar{b}}{}^{c}T_c + f_{a\bar{b}}{}^{\bar{c}}T_{\bar{c}}$, $Tr(T_aT_b) = 0$, $Tr(T_aT_{\bar{b}}) = \delta_{a\bar{b}}$ where f's are the structure constants.

Then the $N = 2$ currents $\mathcal{Q}^a$ and $\mathcal{Q}^{\bar{a}}$ can be characterized by the *nonlinear* constraints. We will only discuss the *"chiral"* currents in the sense that they are annihilated by D_- and $\overline{D}_-$. ($D_-\mathcal{Q}^a = \overline{D}_-\mathcal{Q}^a = D_-\mathcal{Q}^{\bar{a}} = \overline{D}_-\mathcal{Q}^{\bar{a}} = 0$.) For brevity we will write D for D_+ and $\overline{D}$ for $\overline{D}_+$.

$$D\mathcal{Q}^a = -\frac{1}{2(k+3)} f^a{}_{bc}\mathcal{Q}^b\mathcal{Q}^c, \quad \overline{D}\mathcal{Q}^{\bar{a}} = -\frac{1}{2(k+3)} f^{\bar{a}}{}_{\bar{b}\bar{c}}\mathcal{Q}^{\bar{b}}\mathcal{Q}^{\bar{c}}. \tag{2}$$

Here, we work with complex spinor covariant derivatives

$$D = \frac{\partial}{\partial\theta} - \frac{1}{2}\bar{\theta}\partial, \quad \overline{D} = \frac{\partial}{\partial\bar{\theta}} - \frac{1}{2}\theta\partial \tag{3}$$

satisfying the algebra

$$\{D, \overline{D}\} = -\partial \ (= -\partial_z), \tag{4}$$

all other anticommutators vanish*. The dual Coxeter number of $SU(3)$, $\tilde{h}$, is replaced by 3 in the eq. (2) and the f's are antisymmetric in lower two indices†. Any indices can be raised and lowered using $\delta^{a\bar{b}}$and $\delta_{a\bar{b}}$. The fundamental OPE's of these superfields are

$$\begin{aligned}
\mathcal{Q}^a(Z_1)\mathcal{Q}^b(Z_2) &= -\frac{\bar{\theta}_{12}}{z_{12}} f^{ab}{}_c\mathcal{Q}^c - \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k+3} f^a{}_{ec} f^{be}{}_d \mathcal{Q}^c\mathcal{Q}^d \\
\mathcal{Q}^{\bar{a}}(Z_1)\mathcal{Q}^{\bar{b}}(Z_2) &= -\frac{\theta_{12}}{z_{12}} f^{\bar{a}\bar{b}}{}_{\bar{c}}\mathcal{Q}^{\bar{c}} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\frac{1}{k+3} f^{\bar{a}}{}_{\bar{e}\bar{c}} f^{\bar{b}\bar{e}}{}_{\bar{d}} \mathcal{Q}^{\bar{c}}\mathcal{Q}^{\bar{d}} \\
\mathcal{Q}^a(Z_1)\mathcal{Q}^{\bar{b}}(Z_2) &= \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{1}{2}\left[(k+3)\delta^{a\bar{b}} + f^a{}_{cd} f^{\bar{b}cd}\right] - \frac{1}{z_{12}}(k+3)\delta^{a\bar{b}} \\
&\quad -\frac{\theta_{12}}{z_{12}} f^{a\bar{b}}{}_c\mathcal{Q}^c - \frac{\bar{\theta}_{12}}{z_{12}} f^{a\bar{b}}{}_{\bar{c}}\mathcal{Q}^{\bar{c}} \\
&\quad -\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[f^{a\bar{b}}{}_c\bar{D}\mathcal{Q}^c + \frac{1}{k+3} f^{a\bar{c}}{}_d f^{\bar{b}}{}_{\bar{c}\bar{e}} \mathcal{Q}^d\mathcal{Q}^{\bar{e}}\right],
\end{aligned} \tag{5}$$

where

$$\theta_{12} = \theta_1 - \theta_2, \ \bar{\theta}_{12} = \bar{\theta}_1 - \bar{\theta}_2, \ z_{12} = z_1 - z_2 + \frac{1}{2}(\theta_1\bar{\theta}_2 + \bar{\theta}_1\theta_2) \tag{6}$$

and all the superfields in the right hand side are evaluated at Z_2. As we will see in later, these OPE's can be used as the basis for a generalized Sugawara construction and coset construction. It can be checked that the Jacobi identities of this algebra are satisfied only if

*Notice that we take different conventions from those of ref. [9]

†We choose the structure constants as follows: $f_{14}{}^{3} = f_{3\bar{4}}{}^{1} = f_{4\bar{3}}{}^{\bar{1}} = -1$, $f_{1\bar{1}}{}^{\bar{2}} = f_{1\bar{2}}{}^{1} = -i$, $f_{23}{}^{3} = f_{3\bar{3}}{}^{\bar{2}} = f_{4\bar{4}}{}^{2} = -\frac{1}{2}(\sqrt{3}+i)$, $f_{24}{}^{4} = f_{3\bar{3}}{}^{2} = f_{4\bar{4}}{}^{\bar{2}} = -\frac{1}{2}(\sqrt{3}-i)$.

we have the above nonlinear constraints (2). The quadratic terms in (5) are essential for this consistency. Of course, the familiar reduction to $N=1$ unconstrained superfield description of (5) has been discussed already in [9].

Let us now focus on the $N=2$ superconformal algebra. The appropriate generalization to $N=2$ superspace of the well-known Sugawara construction gives the following formula for the $N=2$ stress tensor in terms of the supercurrents $\mathcal{Q}^a$ and $\mathcal{Q}^{\bar{a}}$ ([8]),

$$\begin{aligned}
\mathcal{T}_G &= -\frac{1}{k+3}\delta_{a\bar{b}}\mathcal{Q}^a\mathcal{Q}^{\bar{b}} + \frac{1}{k+3}(\delta_{b\bar{c}}f_{\bar{a}}{}^{b\bar{c}}D\mathcal{Q}^{\bar{a}} + \delta_{b\bar{c}}f_a{}^{b\bar{c}}\overline{D}\mathcal{Q}^a) \\
&= -\frac{1}{k+3}\left[\mathcal{Q}^1\mathcal{Q}^{\bar{1}} + \mathcal{Q}^2\mathcal{Q}^{\bar{2}} + \mathcal{Q}^3\mathcal{Q}^{\bar{3}} + \mathcal{Q}^4\mathcal{Q}^{\bar{4}} - (\sqrt{3}-i)D\mathcal{Q}^{\bar{2}} - (\sqrt{3}+i)\overline{D}\mathcal{Q}^2\right]. \quad (7)
\end{aligned}$$

It satisfies the OPE according to (5),

$$\mathcal{T}_G(Z_1)\mathcal{T}_G(Z_2) = \frac{1}{z_{12}^2}\frac{c_G}{3} + \left[\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} - \frac{\theta_{12}}{z_{12}}D + \frac{\bar{\theta}_{12}}{z_{12}}\overline{D} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial_2\right]\mathcal{T}_G\,. \quad (8)$$

The central charge is

$$c_G = \frac{3\,\dim G}{2}\left[1 - \frac{2\tilde{h}}{3(k+\tilde{h})}\right] = 12\frac{k+1}{k+3}. \quad (9)$$

The $N=2$ superfield $\mathcal{T}_G$ has as its component fields the bosonic stress tensor T of spin 2, two supercurrents G^+ and G^- of spins 3/2 and the $U(1)$ current J of spin 1, which together form the familiar $N=2$ current algebra.

Let $H = SU(2)\times U(1)$ be a subgroup of $G(=SU(3))$ and denote the indices corresponding to H and the coset $\frac{G}{H} = \frac{SU(3)}{SU(2)\times U(1)}$ by $1,\bar{1},2,\bar{2}$ and $3,\bar{3},4,\bar{4}$, respectively. Then $N=2$ currents can be divided into two sets. We may construct another $N=2$ superconformal algebra,

$$\mathcal{T}_H = -\frac{1}{k+3}\left[\mathcal{Q}^1\mathcal{Q}^{\bar{1}} + \mathcal{Q}^2\mathcal{Q}^{\bar{2}} + iD\mathcal{Q}^{\bar{2}} - i\overline{D}\mathcal{Q}^2\right]. \quad (10)$$

It satisfies the OPE

$$\mathcal{T}_H(Z_1)\mathcal{T}_H(Z_2) = \frac{1}{z_{12}^2}\frac{c_H}{3} + \left[\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} - \frac{\theta_{12}}{z_{12}}D + \frac{\bar{\theta}_{12}}{z_{12}}\overline{D} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial_2\right]\mathcal{T}_H\,. \quad (11)$$

The total central charge is the sum of contributions $\frac{3\,\dim H}{2}\left[1-\frac{2\tilde{h}}{3(k+\tilde{h})}\right]$ for each simple factor of H. That is,

$$c_H = \frac{9}{2}\left[1 - \frac{4}{3(k+2)}\right] + \frac{3}{2} = 6\frac{k+2}{k+3}. \quad (12)$$

It is easy to see that a realization of 'large' $N=4$ superconformal algebra [10] with $k_+ = k+2$, $k_- = 1$ where the two affine $SU(2)$ subalgebras have level k_+ and k_- respectively, can be generated by $\{\mathcal{Q}^{2,\bar{2}},\ \mathcal{Q}^1,\ \mathcal{Q}^{\bar{1}},\ \mathcal{T}_H\}$ as shown in [9].

We take a closer look at the supersymmetric coset models based on $N=2$ CP_2 model with level k of $SU(3)$. Let us define

$$\mathcal{T}_{\frac{G}{H}} = \mathcal{T}_G - \mathcal{T}_H = -\frac{1}{k+3}\left[\mathcal{Q}^3\mathcal{Q}^{\bar{3}} + \mathcal{Q}^4\mathcal{Q}^{\bar{4}} - \sqrt{3}D\mathcal{Q}^{\bar{2}} - \sqrt{3}\overline{D}\mathcal{Q}^2\right]. \tag{13}$$

Using the eqs. (5), we can conclude that

$$\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{Q}^1(Z_2) = \mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{Q}^{\bar{1}}(Z_2) = \mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{Q}^2(Z_2) = \mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{Q}^{\bar{2}}(Z_2) = 0 \tag{14}$$

and then $\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{T}_H(Z_2) = 0$. This decomposition implies from (8) and (11) that $\mathcal{T}_{\frac{G}{H}}$ satisfies the OPE

$$\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{T}_{\frac{G}{H}}(Z_2) = \frac{1}{z_{12}^2}\frac{c_{\frac{G}{H}}}{3} + \left[\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2} - \frac{\theta_{12}}{z_{12}}D + \frac{\bar{\theta}_{12}}{z_{12}}\overline{D} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial_2\right]\mathcal{T}_{\frac{G}{H}}, \tag{15}$$

where

$$c_{\frac{G}{H}} = c_G - c_H = 6\frac{k}{k+3} \tag{16}$$

which is the same as (1) (as a function of k). The super currents $\{\mathcal{Q}^3, \mathcal{Q}^{\bar{3}}, \mathcal{Q}^4, \mathcal{Q}^{\bar{4}}\}$ are not $N=2$ primary fields because of nonlinear constraints among them. For example,

$$\begin{aligned}
&\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{Q}^3(Z_2) = \\
&\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\frac{k}{2(3+k)}\mathcal{Q}^3 + \frac{1}{z_{12}}\frac{k}{(3+k)}\mathcal{Q}^3 + \frac{\bar{\theta}_{12}}{z_{12}}\left[\overline{D}\mathcal{Q}^3 + \frac{(i-\sqrt{3})}{2(3+k)}\mathcal{Q}^{\bar{2}}\mathcal{Q}^3\right] \\
&+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[\frac{1}{(3+k)^2}\mathcal{Q}^1\mathcal{Q}^3\mathcal{Q}^{\bar{1}} + \frac{i}{(3+k)^2}\mathcal{Q}^{\bar{2}}\mathcal{Q}^1\mathcal{Q}^4\right. \\
&\left.+\frac{1}{(3+k)}\overline{D}\mathcal{Q}^1\mathcal{Q}^4 + \frac{(i+\sqrt{3})}{2(3+k)}\overline{D}\mathcal{Q}^2\mathcal{Q}^3 + \frac{(-i+\sqrt{3})}{2(3+k)}D\mathcal{Q}^{\bar{2}}\mathcal{Q}^3 + \partial\mathcal{Q}^3\right].
\end{aligned} \tag{17}$$

And the $U(1)$ charge is the fractional function of k, $k/(3+k)$. However we do not write dowm explicitly for other OPE's $\mathcal{T}_{\frac{G}{H}}$ with $\{\mathcal{Q}^{\bar{3}}, \mathcal{Q}^4, \mathcal{Q}^{\bar{4}}\}$ here, they are necessary for (19).

In order to extend the coset construction to the higer spin current we proceed as follows [3]. Sugawara's expression for the higher spin current can be obtained as composite operators of currents contracted with δ and f tensors. Notice that each term should have the correct $U(1)$ charge conservation, that is, $\mathcal{W}_{\frac{G}{H}}$ has vanishing $U(1)$ charge. The dimension 2 coset field $\mathcal{W}_{\frac{G}{H}}$ is uniquely fixed by the requirements that it should be a superprimary field of dimension 2 with respect to $\mathcal{T}_{\frac{G}{H}}$ and the OPE's with $\{\mathcal{Q}^1, \mathcal{Q}^{\bar{1}}, \mathcal{Q}^2, \mathcal{Q}^{\bar{2}}\}$ are regular up to overall constant $A(k)$:

$$\mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{Q}^1(Z_2) = \mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{Q}^{\bar{1}}(Z_2) = \mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{Q}^2(Z_2) = \mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{Q}^{\bar{2}}(Z_2) = 0, \tag{18}$$

$$\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2) = \left[\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}2 - \frac{\theta_{12}}{z_{12}}D + \frac{\bar{\theta}_{12}}{z_{12}}\overline{D} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\partial_2\right]\mathcal{W}_{\frac{G}{H}}. \tag{19}$$

Thus

$$\begin{aligned}
\mathcal{W}_{\frac{G}{H}} \;=\; & A(k)(3+k)\left[\frac{k(3+k)^2(5+k)}{2(3-5k)}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} + \frac{(3+k)^2}{2}[D,\overline{D}]\mathcal{T}_H - 2\mathcal{Q}^2\mathcal{Q}^{\bar{2}}\mathcal{Q}^4\mathcal{Q}^{\bar{4}}\right.\\
& +(i+\sqrt{3})\mathcal{Q}^2\mathcal{Q}^{\bar{2}}D\mathcal{Q}^{\bar{2}} - i\mathcal{Q}^2\mathcal{Q}^3\mathcal{Q}^{\bar{1}}\mathcal{Q}^{\bar{4}} + (-i+\sqrt{3})\mathcal{Q}^2\overline{D}\mathcal{Q}^2\mathcal{Q}^{\bar{2}}\\
& +(3+k)\mathcal{Q}^2\partial\mathcal{Q}^{\bar{2}} + i\mathcal{Q}^{\bar{2}}\mathcal{Q}^1\mathcal{Q}^4\mathcal{Q}^{\bar{3}} - (3+k)(5+k)\mathcal{Q}^4\partial\mathcal{Q}^{\bar{4}}\\
& -(3+k)(5+k)\mathcal{T}_{\frac{G}{H}}\mathcal{Q}^4\mathcal{Q}^{\bar{4}} + \frac{3k(3+k)^2(5+k)}{2(3-5k)}\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}} - (3+k)^2\mathcal{T}_{\frac{G}{H}}\mathcal{T}_H\\
& +\frac{(3+k)(7-3i\sqrt{3}+k-i\sqrt{3}k)}{(-i+\sqrt{3})}\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{Q}^2 + \frac{2(3+k)(4-i\sqrt{3}+k)}{(-i+\sqrt{3})}\mathcal{T}_{\frac{G}{H}}D\mathcal{Q}^{\bar{2}}\\
& -(3+k)\mathcal{T}_{\frac{G}{H}}\mathcal{Q}^2\mathcal{Q}^{\bar{2}} - 2(3+k)\mathcal{T}_H\mathcal{Q}^4\mathcal{Q}^{\bar{4}} + \frac{(3+k)^2}{2}\mathcal{T}_H\mathcal{T}_H\\
& +(-i+\sqrt{3})(3+k)\mathcal{T}_H\overline{D}\mathcal{Q}^2 + (i+\sqrt{3})(3+k)\mathcal{T}_H D\mathcal{Q}^{\bar{2}} + (3+k)\mathcal{T}_H\mathcal{Q}^2\mathcal{Q}^{\bar{2}}\\
& +(3+k)\overline{D}\mathcal{Q}^1\mathcal{Q}^4\mathcal{Q}^{\bar{3}} + (-i+5\sqrt{3}-ik+\sqrt{3}k)\overline{D}\mathcal{Q}^2\mathcal{Q}^4\mathcal{Q}^{\bar{4}}\\
& -\frac{(-5i+3\sqrt{3}-2ik)}{(-i+\sqrt{3})}\overline{D}\mathcal{Q}^2\overline{D}\mathcal{Q}^2 - (5+k)\overline{D}\mathcal{Q}^2 D\mathcal{Q}^{\bar{2}}\\
& +(3+k)D\mathcal{Q}^{\bar{1}}\mathcal{Q}^3\mathcal{Q}^{\bar{4}} + (i+5\sqrt{3}+ik+\sqrt{3}k)D\mathcal{Q}^{\bar{2}}\mathcal{Q}^4\mathcal{Q}^{\bar{4}}\\
& -\frac{(-2i+4\sqrt{3}+ik+\sqrt{3}k)}{(-i+\sqrt{3})}D\mathcal{Q}^{\bar{2}}D\mathcal{Q}^{\bar{2}} - (3+k)\partial\mathcal{Q}^2\mathcal{Q}^{\bar{2}}\\
& \left.+(3+k)(5+k)\partial\mathcal{Q}^4\mathcal{Q}^{\bar{4}}\right].
\end{aligned} \tag{20}$$

In the above expression, we used the fact that the normal ordering of $\mathcal{Q}^3\mathcal{Q}^{\bar{3}}$ can be written as $-(k+3)\mathcal{T}_{\frac{G}{H}}$ besides other terms, seen by (13) and similarly ($\mathcal{Q}^1\mathcal{Q}^{\bar{1}}$ as $-(k+3)\mathcal{T}_H$) because in the OPE $\mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2)$, $\mathcal{T}_{\frac{G}{H}}(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2)$ transforms as (19) and $\mathcal{T}_H(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2) = 0$ which make the calculations easier. The coefficients of composite operators having $1, \bar{1}, 2, \bar{2}$ indices in (20) can not fixed as (19) but as (18) because $\mathcal{T}_{\frac{G}{H}}$ commutes with $\{\mathcal{Q}^1, \mathcal{Q}^{\bar{1}}, \mathcal{Q}^2, \mathcal{Q}^{\bar{2}}\}$ as (14). All the $D\mathcal{Q}^a, \overline{D}\mathcal{Q}^{\bar{a}}$'s are written as the right hand side of (2). By requiring the fourth order pole of $\mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2)$ should be $c_{\frac{G}{H}}/2$, the normalization factor $A(k)$ can be fixed as follows:

$$A(k) = \sqrt{\frac{3(-3+5k)}{(-1+k)(k+3)^6(k+5)(2k+3)}}. \tag{21}$$

After a tedious calculation by writing $\mathcal{W}_{\frac{G}{H}}(Z_1)$ as in (20) and computing the 29th OPE's

with $\mathcal{W}_{\frac{G}{H}}(Z_2)$, we arrive at the final result,

$$
\begin{aligned}
&\mathcal{W}_{\frac{G}{H}}(Z_1)\mathcal{W}_{\frac{G}{H}}(Z_2) = \\
&\frac{1}{z_{12}^4}\frac{3k}{(3+k)} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^4}3\mathcal{T}_{\frac{G}{H}} + \frac{\bar{\theta}_{12}}{z_{12}^3}3\overline{D}\mathcal{T}_{\frac{G}{H}} - \frac{\theta_{12}}{z_{12}^3}3D\mathcal{T}_{\frac{G}{H}} + \frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^3}3\partial\mathcal{T}_{\frac{G}{H}} \\
&+\frac{1}{z_{12}^2}\left[2\alpha\mathcal{W}_{\frac{G}{H}} + \frac{6k}{(3-5k)}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} + \frac{3(3+k)}{(3-5k)}\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}\right] \\
&+\frac{\bar{\theta}_{12}}{z_{12}^2}\left[\alpha\overline{D}\mathcal{W}_{\frac{G}{H}} + \frac{9(-1+k)}{(-3+5k)}\partial\overline{D}\mathcal{T}_{\frac{G}{H}} + \frac{3(3+k)}{(3-5k)}\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{T}_{\frac{G}{H}}\right] \\
&+\frac{\theta_{12}}{z_{12}^2}\left[\alpha D\mathcal{W}_{\frac{G}{H}} + \frac{9(1-k)}{(-3+5k)}\partial D\mathcal{T}_{\frac{G}{H}} + \frac{3(3+k)}{(3-5k)}\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}}\right] \\
&+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}^2}\left[\frac{\alpha(12+k)}{6(-2+k)}[D,\overline{D}]\mathcal{W}_{\frac{G}{H}} + \frac{7\alpha(3+k)}{3(-2+k)}\mathcal{T}_{\frac{G}{H}}\mathcal{W}_{\frac{G}{H}}\right. \\
&+\frac{9}{2}\beta(-2+k)k(3+k)\partial[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} + \frac{3}{2}\beta(3+k)(6+k+5k^2)\mathcal{T}_{\frac{G}{H}}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} \\
&+\frac{3}{2}\beta(3+k)^2(1+3k)\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}} + 18\beta(-2+k)k(3+k)\overline{D}\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}} \\
&\left.-\frac{3}{2}\beta(9-6k-3k^2+8k^3)\partial^2\mathcal{T}_{\frac{G}{H}}\right] \\
&+\frac{1}{z_{12}}\left[\frac{3k}{(3-5k)}\partial[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} + \frac{3(3+k)}{(3-5k)}\partial\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}} + \alpha\partial\mathcal{W}_{\frac{G}{H}}\right] \\
&+\frac{\bar{\theta}_{12}}{z_{12}}\left[\frac{\alpha(-3+5k)}{3(2-k)(1+k)}\partial\overline{D}\mathcal{W}_{\frac{G}{H}} - \frac{3}{4}\beta k(9+12k+11k^2)\partial^2\overline{D}\mathcal{T}_{\frac{G}{H}}\right. \\
&+\frac{\alpha(3+k)(5+k)}{3(-2+k)(1+k)}\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{W}_{\frac{G}{H}} + \frac{3}{2}\beta(3+k)^2(1+3k)\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{T}_{\frac{G}{H}} \\
&+\frac{\alpha(3+k)(-3+5k)}{3(-2+k)(1+k)}\overline{D}\mathcal{T}_{\frac{G}{H}}\mathcal{W}_{\frac{G}{H}} + \frac{3k(3+k)}{2(1-k)(3+2k)}\overline{D}\mathcal{T}_{\frac{G}{H}}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}} \\
&\left.+9\beta(3+k)(-1+2k+k^2)\partial\overline{D}\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}} + \frac{3(3+k)}{2(-1+k)(3+2k)}\partial\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{T}_{\frac{G}{H}}\right] \\
&+\frac{\theta_{12}}{z_{12}}\left[\frac{\alpha(-3+5k)}{3(2-k)(1+k)}\partial D\mathcal{W}_{\frac{G}{H}} + \frac{9}{2}\beta k(3+k^2)\partial^2 D\mathcal{T}_{\frac{G}{H}}\right. \\
&+\frac{\alpha(3+k)(5+k)}{3(2-k)(1+k)}\mathcal{T}_{\frac{G}{H}}D\mathcal{W}_{\frac{G}{H}} - \frac{3}{2}\beta(3+k)^2(1+3k)\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}} \\
&+\frac{3k(3+k)}{2(-1+k)(3+2k)}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}} + \frac{\alpha(3+k)(-3+5k)}{3(2-k)(1+k)}D\mathcal{T}_{\frac{G}{H}}\mathcal{W}_{\frac{G}{H}}
\end{aligned}
$$

$$
+9\beta(3+k)(-1+2k+k^2)\partial D\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}+\frac{3(3+k)}{2(-1+k)(3+2k)}\partial\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}}\Bigg]
$$

$$
+\frac{\theta_{12}\bar{\theta}_{12}}{z_{12}}\left[\frac{\alpha k(5+k)}{3(-2+k)(1+k)}\partial[D,\overline{D}]\mathcal{W}_{\frac{G}{H}}+\frac{2\alpha(3+k)(3+2k)}{3(-2+k)(1+k)}\mathcal{T}_{\frac{G}{H}}\partial\mathcal{W}_{\frac{G}{H}}\right.
$$

$$
+\frac{2\alpha(3+k)}{3(1+k)}\overline{D}\mathcal{T}_{\frac{G}{H}}D\mathcal{W}_{\frac{G}{H}}+12\beta(-2+k)k(3+k)\partial\overline{D}\mathcal{T}_{\frac{G}{H}}D\mathcal{T}_{\frac{G}{H}}
$$

$$
+3\beta(3+k)(3+k^2)\partial[D,\overline{D}]\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}+\frac{2\alpha(3+k)}{3(1+k)}D\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{W}_{\frac{G}{H}}
$$

$$
-12\beta(-2+k)k(3+k)\partial D\mathcal{T}_{\frac{G}{H}}\overline{D}\mathcal{T}_{\frac{G}{H}}+\frac{2\alpha(3+k)(1+3k)}{3(-2+k)(1+k)}\partial\mathcal{T}_{\frac{G}{H}}\mathcal{W}_{\frac{G}{H}}
$$

$$
+3\beta k(3+k)(1+3k)\partial\mathcal{T}_{\frac{G}{H}}[D,\overline{D}]\mathcal{T}_{\frac{G}{H}}+3\beta(3+k)^2(1+3k)\partial\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}\mathcal{T}_{\frac{G}{H}}
$$

$$
\left.-\frac{3}{2}\beta(-3-4k-3k^2+2k^3)\partial^3\mathcal{T}_{\frac{G}{H}}\right] \tag{22}
$$

where

$$
\alpha=\frac{3\sqrt{3}(2-k)(1+k)}{\sqrt{(-1+k)(5+k)(3+2k)(-3+5k)}},\quad \beta=\frac{1}{(3-5k)(-1+k)(3+2k)} \tag{23}
$$

Therefore the algebra, (15), (19), and (22) coincide with exactly those explained in [7, 11] with all superfields replaced by their coset analogues. From the explicit OPE's about $N=2$ current satisfying nonlinear constraints in $SU(3)$ supersymmetric WZW model, we have determined spin 2 supermultiplet by exploiting a generalized Sugawara construction and coset construction in $N=2$ superspace. For the central charge less than six of any unitary representation of this algebra consisting of $\{\mathcal{T}_{\frac{G}{H}},\mathcal{W}_{\frac{G}{H}}\}$, we have found the full algebraic structure of it. We realize that our findings from the viewpoint of supersymmetric WZW model in third approach mentioned in the introduction reproduce those [7, 11] in the first approach. An interesting aspect to study in the future direction is how to realize for the noncompact coset $SU(2,1)/U(2)$ model which has the central charge $c=6(1+\frac{3}{k+2})\ \ k=1,2,\cdots$ that contains $c=9$.

It would be very interesting to understand how our explicit form of $N=2$ W_3 symmetry current enables us to a basis for the Landau-Ginzberg polynomials [12] and how to generalize our results for CP_3 Kazama-Suzuki model [13] in which they constructed the nonlinear extension of $N=2$ superconformal algebra by two superprimary fields of spin $2,3$ with zero $U(1)$ charge and even CP_n case. For CP_3, we have odd dimensional groups $G=SU(4)$ and $H=SU(3)\times U(1)$ then maybe we can construct on the even dimensional groups $G=SU(4)\times U(1)$ and $H=SU(3)\times U(1)\times U(1)$ by following the same procedure. From the viewpoint of second approach in [1] it was worked out that $N=2$ coset model $CP_n=\frac{SU(n+1)}{SU(n)\times U(1)}$ can be obtained by the quantum hamiltonian reduction of the affine Lie superalgebra $A(n,n-1)^{(1)}$ and the chiral algebra of this model is $N=2$ W- algebra via the quantum super Miura transformation [14, 15, 16]. It is also open problem to study how

our coset W algebras are related to quantum Drinfeld-Sokolov W algebras in the spirit of [14, 15, 16, 17].

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